A bound on the cohomology of quasiregularly elliptic manifolds

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A classical result gives that if there exists a holomorphic mapping $f\colon \mathbb{C} \to M$, then M is homeomorphic to $\mathbb{S}^2$ or $\mathbb{S}^1 \times \mathbb{S}^1$, where M is a compact Riemann surface. I will discuss a generalization of this problem to higher dimensions. I will show that if M is a d-dimensional, closed, connected, orientable Riemannian manifold that admits a quasiregular mapping from $\mathbb{R}^d$, then the dimension of the degree l de Rham cohomology of M is bounded above by $\binom{d}{l}$. This is a sharp upper bound that proves a conjencture by Bonk and Heinonen. A corollary of this theorem answers an open problem posed by Gromov. He asked whether there exists a simply connected manifold that does not admit a quasiregular map from $\mathbb{R}^d$. The result gives an affirmative answer to this question.