Modulus and perimeter

Olli Martio
University of Helsinki

A set $E \subset \mathbf{R}^n$ has finite perimeter if the function χ_E has bounded variation, i.e. $\chi_E \in L^1(\mathbf{R}^n)$ and

$$P(E) = \sup\{\int_{\mathbf{R}^n} \chi_E \nabla \cdot \varphi \, dx : \, |\varphi| \leq 1, \, \varphi \in C_0(\mathbf{R}^n, \mathbf{R}^n)\} < \infty.$$

Let Γ be a family of paths in $\mathbf{R}^n$. The AM–modulus of Γ is defined as

$$AM(\Gamma) = \inf\{\liminf_{i\to\infty} \int_{\mathbf{R}^n} \rho_i \, dx\}$$

where the infimum is taken over all sequences of Borel functions $\rho_i \geq 0$ such that $\liminf_{i\to\infty} \int_\gamma \rho_i \, ds \geq 1$ for all $\gamma \in \Gamma$. The perimeter of a Lebesgue measurable set E with $m_n(E) < \infty$ has the following (geometric?) characterizations in terms of the AM–modulus of two path families intimately connected to te measure theoretic properties of E:

$$AM(\Gamma^c_{\partial_* E}) = P(E) = AM(\Gamma_{cross}(E))$$

where $\Gamma^c_{\partial_* E}$ is the family of all paths $\gamma : [a, b] \to \mathbf{R}^n$ which meet the measure theoretic boundary $\partial_* E$ of E at an interior point of $[a, b]$ and $\Gamma_{cross}(E)$ is the family of all paths which join the measure theoretic exterior to the measure theoretic interior of E.

The results are joint work with V. Honzlová Exnerová and J. Malý.