Variational approach to quasiconformal geometry and nonlinear elasticity

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This is an overview of recent joint work with Jani Onninen. Energy-minimal configurations in mathematical models of Nonlinear Elasticity (NE) are profoundly insightful for Geometric Function Theory (GFT) through variational integrals. The energy-integrals are subjected to Sobolev homeomorphisms and their weak and strong limits. For example, a little reflection on Teichmüller theory of quasiconformal deformations encourages to study mappings of smallest $\mathcal{L}^p$-norm of the distortion functions, referred to as $\mathcal{L}^p$-Grötzsch extremals. We shall illustrate the concept of free-Lagrangians as a tool for solving other extremal problems. Conversely, Hopf quadratic differentials in planar GFT carry mysteries of a lot of things in mathematical models of elastic deformations, such as fractures and formation of cracks. Cracks propagate along vertical trajectories of a Hopf differential; a phenomenon that should appeal to researchers in the engineering fields as well.