Quasiconformal mappings and the skew of triangles

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The skew of a topological triangle is the ratio between the largest and smallest Euclidean distances between any two distinct vertices (thus only the vertices matter, not the shape of the sides). This concept was introduced by John Hubbard who proved that if, for a homeomorphism between plane domains, the skew of the image triangle is bounded for all Euclidean triangles in the domain of definition with skew below a certain absolute constant (about 1.53), then the mapping is quasiconformal. Hubbard asked whether it suffices to consider only equilateral triangles. Javier Aramayona and Peter Haïssinsky showed that there exists a constant $\varepsilon > 0$ such that it is sufficient that the skew of the image of every equilateral triangle is at most $1 + \varepsilon$. In joint work with Colleen Ackermann and Haïssinsky, we have proved Hubbard's conjecture, that is, the mapping is quasiconformal if there is a uniform bound for the skew of the image of every equilateral triangle (since quasiconformality is a local property, it suffices to consider triangles contained in small neighbourhoods of points of the domain). It is possible to base the proof on the metric or the analytic definition of quasiconformality. We also obtain a bound for the maximal dilatation of the mapping in terms of a suitable local bound for the skew.