Intrinsic graphs, singular integrals, and Lipschitz harmonic functions in the Heisenberg group

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The Heisenberg group $\mathbb{H}^1$ is a Lie group that can be endowed with a left-invariant distance d closely connected to a second order PDE, the sub-Laplace equation. Solutions to this equation are the harmonic functions in $\mathbb{H}^1$. In analogy with $\mathbb{R}^n$, a closed set $E \subset \mathbb{H}^1$ is said to be removable for Lipschitz harmonic functions if for every open set $D \supseteq E$, every Lipschitz function $f : D \to \mathbb{R}$ that is harmonic in $D \setminus E$, is in fact harmonic in D. Examples of Ahlfors regular sets of codimension 1 in $\mathbb{H}^1$ that are removable for Lipschitz harmonic functions were constructed by V. Chousionis and P. Mattila. In this talk, which is based on joint work with V. Chousionis and T. Orponen, I will present surfaces - certain intrinsic graphs – that are *not* removable for Lipschitz harmonic functions. The non-removability is connected to the L^2 boundedness of the Heisenberg Riesz transform on the respective set. I will discuss this transform as an instance of a singular integral operator whose kernel has good cancellation properties for the problem at hand, even if it is not antisymmetric.
