Noncollapsed Gromov-Hausdorff limit spaces with Ricci curvature bounded below

Jeff Cheeger

Courant Institute of Mathematical Sciences

We will discuss recent work with Aaron Naber and Wenshuai Jiang on the structure of Gromov-Hausdorff limit spaces $(M_i^n, g_i, p_i) \stackrel{d_{GH}}{\longrightarrow} (X^n, d, p)$, with $\mathrm{Ric}_{M_i^n} \geq -(n-1)$ and $\mathrm{Vol}(B_1(p_i)) \geq \mathrm{v} > 0$. By definition, the singular set S is the set of points for which no tangent cone is isometric to Euclidean space $\mathbb{R}^n$. Elementary examples show that S need not be closed. By work of Cheeger-Colding, the Hausorff dimension of S satisfies the sharp bound $\mathrm{dim} S \leq n-2$. We show in particular there is a filtration by closed $(n-2)$-rectifiable subsets, $S = \bigcup_{\epsilon > 0} S_\epsilon$, such that $\mathcal{R}_\epsilon := X^n \setminus S_\epsilon$ is bi-Hölder to a smooth Riemannian manifold and the Hausdorff $(n-2)$-dimensional Hausdorff measure of S_ϵ satisfies $\mathcal{H}^{n-2}(S_\epsilon \cap B_1(p)) \leq c(n, \mathrm{v}, \epsilon)$. These results follow from stronger results on the *quantitative stratification*, which in turn follow from results on *neck regions* and *neck decompositions*.
